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A Hybrid Intelligent Optimization Method for Multiple Metal Grades Optimization

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Abstract: One of the most important aspects of metal mine design is to determine the optimum cut-off grades and milling grades which relate to the economic efficiency of enterprises and the service life of mines. This paper proposes a hybrid intelligent framework which is based on stochastic simulations and regression, artificial neural network and genetic algorithms is employed for grade optimization. Firstly, stochastic simulation and regression are used to simulate the uncertainty relations between cut-off grade and the loss rate.Secondly,BP and RBF network are applied to establish two complex relationships from the four variables of cut-off grade, milling grade, geological grade and recoverable reserves to lost rate and total cost, respectively, in which, BP is used for the one of lost rate, and RBF is for the other. Meanwhile, the real-coding genetic algorithm is performed to search the optimal grades (cut-off grade and milling grade) and the weights of neural networks globally. Finally, the model has been applied to optimize grades of Daye Iron Mine. The results show there are 6. 6978 milling Yuan added compare to un-optimized grades.

Keywords:multiplemetal grades; cut-off grade; hybrid intelligent; artificial neural networks; genetic algorithms;optimization

1. Introduction

The cut-off grade and millinggrade are two crucial indices of mining process. Cut-off grade is defined as the grade that is used to discriminate between ore and wastewithin a given ore body [1,2]. If material grade in the mineral deposit isabove cut-off grade it is classified as ore and if material grade is below cut-off grade it isclassified as waste. While milling grade refers to the minimum average grade on usable deposit in the industry, namely, on the condition of the recent technical economy, to exploit the technically practical and economically reasonableminimum grade. Specifically, the lower of these two parameters will

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increase the production cost of ore and further influence production capacity. When these values are too high, there will be a loss of ore resources and a reduction in the longevity of mine[3]. Therefore, theoptimalchoices of cut-off grade and millingarecrucialto enterprise economics and resource sustainability.

Many authors have proposedvarious concepts and calculation methods regarding the cut-off grade and milling grade from different perspectives. Different approaches can be categorized as follows:(a) statistical estimation [4], inwhich classical statistical methods are used to estimate the grades. (b) dynamicprogrammingoptimize model [5,6,7]，in which a dynamic programming model is developed to optimize cut-off grade according to the different stages of metal mining and the factors affecting grades.Asadproposes a new cut-off optimization algorithm which considers the dynamic metal price and cost escalation during mine life[8].(c)the generalized reduced gradient algorithms[9]，which describes the determination of a cut-off grade strategy basedon Lane’s algorithm by adding an optimization factor based on thegeneralized reduced gradient algorithm to maximize theproject's NPV.(d) Staticoptimization based on Lane's theory [10, 11].These methods are good for specificmine production and management, but suffer in their generalizability with the low robustness. Until now, the cut-off grade and milling gradeare mainly determined by experimental data or worker’s experiencebecause of the particularity of production environment,the multiplicity of ore drawing and the extension of mine management. Moreover,thesetheories and methodscannot be used widelyby reason of its aim at the specific ore depositand production technique. Alongwith the production development, increase of resources’scarcity,and enhancement of technical and managing level, the experts andmanagers gradually realize that, the determining method basedon workers’experience, is easy and feasible though, greatly increasesthe mining and milling cost, and wastes resourceseriously.

Obviously, cut-off grade and milling grade are the complex variables related to benefit, cost, geological grade, recoverable reserves, loss rate, etc.. In fact, in the mining and milling process, cut-off grade and milling grade are the complex non-linear functions related to above variables along with the change of time and space. It’s difficult to directly obtain their formulas (e.g.analytic functions) or indirectly establish their expressions (e.g.differential equations).Therefore, it is urgent to propose a high nonlinear mapping and intelligent method to optimize the cut-off grade and milling grade. Modelingand operation limited numbers of data is the most important superiority of soft computing compared with other conventional methods used in geosciences such as statistics and geostatistics. Applicability of soft computing inmodelingand operation the geological based domains has been discussed by different authors[12, 13]. One of the soft computingmethods, artificial neural networks (ANNs), has been proposed as an efficient tool for modeling and optimizing in recent years, mainly because of ANNs' wide range of applicability and their capability to treat complicated and non-linear problems [14,15]. But ANN has anevident disadvantage, i.e. training time is long and could be trapped in a local optimum; in addition, the calculation may overflowor fluctuate between the optima. To overcome the disadvantage, many author applied Genetic Algorithms (GA) to form a hybrid method by GA-ANN in which ANN have been constructed and trained to accelerate the GA-basedsearch during the optimization process [16, 17].GA is another soft computing tool,which could provide versatile problem solving mechanism for search, adaptation, and learning in a variety of application domains, especially for those problems in which heuristic methods lead to unsatisfactory results[18]. They are random searches and optimization techniques guided by the principles of evolution and natural genetics. They are efficient, adaptive, and robust search processes, producing near-optimal solutions and having a large amount of implicit parallelism. The combined GA-ANN algorithmhas a great potential to handle problems such as optimizationin complicated nonlinear systems[19]. Although some research focus on applying GA-ANN to optimize the grade in recently years [20, 21], there are still many problems to be further studied such as how to hybrid GA and ANN; how to optimize the grades effectively.

In this paper, a hybrid intelligent framework which is based on stochastic simulations and regression, artificial neural network and genetic algorithms is employed for grades optimization. The idea is detailed as follows. Firstly, stochastic simulation and regressionareused to simulate the uncertainty relations between cut-off grade and the loss rate.Secondly,BP network and RBF network are applied to establish the complex relationship from cut-off grade, milling grade, geologicalgrade and recoverable reserves to lost rate as well as total costs, respectively. Meanwhile, the real-coding genetic algorithm is performed to search the optimal grades (cut-off grade and milling grade) and the weights of neural networks globally. Finally, In order to show the applicability of the hybrid model in grade optimization, the case study of Iron Mine of China presented in the paper.

This paper is organized as follows. Section 2 states the optimalproblem of the multiple metal grades. Section 3 introduces the background of artificial neural network and genetic algorithm. The hybrid intelligent optimization method isdetailedlydescribedin Section 4.Section 5 presents the casestudy ofIron Mine of China. Finally in Section 6, some conclusionsare given.

2. Statement of the multiple metal grades optimizationproblem

The cut-off grade and millinggrade are related with income, cost, and price. These variables vary with time and regions, and are highly complex as well as non-linear.The most common criteria used in cut-offgrade optimization is to maximize the net present value (NPV)[3, 8, 22]).Therefore, we propose a mathematic model for the optimization of the system as follows.

$$\begin{cases} MaxNPV = \sum_{t=1}^{n} \frac{R_t - C(a_t, q_t, a_j, a_r)}{(1+i)^t} \\ s.t \quad R_t = \frac{a_t q_t (1-\phi(a_j)) \cdot \varepsilon(a_t, q_t, a_j, a_r)}{\beta} p \\ \qquad 0 \le a_j \le a_r \le 1 \end{cases} \tag{1}$$

where NPV is the total n monthsnet present value, R_t is the income of the t -th month, a_t is the geographic grade of the t -th month, a_j is the cut-off grade, a_r is the milling grade, q_t is the recoverable reservesthat could be mined in the t -th month, $C(a_t, q_t, a_j, a_r)$ is the mining and selection cost function for the t -th month, $\phi(a_j)$ is the function of lost rate, $\varepsilon(a_t, q_t, a_j, a_r)$ is thefunctionof metal recovery rate, β is the grade of the selected ore, p is the price of the selected ore, and i is the discount rate.

Solving themodel (1) is actually to find an optimal grade combination which makes the NPV be the maximumfrom a variety of grade combinations (cut-off grade, milling grade) in a given data of recoverable reserves, geological grade and unit cost.4 problemsof the solving process are included.

Problem 1: Determining relationship between cut-off grade a_j and loss rate ϕ.

The cut-off grade for ore drawing means the grade of ore in thelast time of ore drawing. The higher cut-off grade leads to the largeramount of waste ore, which makes higher loss rate. The lossrate ϕ depends on the cut-off grade a_j directly, which is shown as follows:

$$\phi = \phi(a_j) \quad (2)$$

From Eq. (2), we could obtain cut-off grade a_j corresponding to acertain loss rate ϕ.

Problem 2: Determining relationship between the metal recovery rate ε and cut-off grade a_j, milling grade a_r, geologicalgrade a_t, and recoverable reserves q_t.In metal recovery rate management, how to accurately forecast the recovery rate is significant for improving the managementlevel of tailing ore.

Present studiesshow that the relatedfactors that influence the recovery rate ε are a_j , a_r , a_t and q_t . Inthe practical production, the relationship between above mentionedfour factors and metal recovery rate is highly complexand nonlinear. We need to establish the following function

$$\varepsilon = \varepsilon(a_t, q_t, a_j, a_r) \tag{3}$$

Problem 3: Determining relationship between the cost of mining& milling C and cut-off grade a_j , milling grade a_r , geologicalgrade a_t , and recoverable reserves q_t .Mining and milling cost can be showed as

$$C = C(a_t, q_t, a_j, a_r) \tag{4}$$

Problem 4: How to efficiently establish above 3 relationships? Obviously, cut-off grade and milling grade are the complex variables related to benefit, cost, geological grade, recoverable reserves, loss rate, etc.. In fact, in the mining and milling process,cut-off grade and milling grade are the complex non-linear functions related to above variables along with the change of time and space. It's difficult to directly obtain their formulas(e.g.analytic functions) orindirectly establish their expressions (e.g.differential equations).

3. Background of artificial neural network and genetic algorithm

3.1 Artificial neural network

Artificial neural network (ANN)theory has been developed in the form of parallel distributednetwork models based on biological learning process of thehuman brain[23].Contrarytothetraditionalmodel-based methods;ANNisadata-driven,self-adaptivemethodinwhich they are fewaprioriassumptionsaboutthemodelsfor problemsinthestudy.Theylearnfromexamplesandcapture subtle functionalrelationshipsamongthedataevenifthe underlyingrelationshipsareunknownordifficulttobedescribed. Thus, ANNiswellsuitedforproblemswhose solutionsrequireknowledge that is difficult to be specified and in which there are not enoughdata or observations. In this sense, they can be treated as one ofthe multivariate nonlinear non-parametric statistical methods.Due to its highly parallel structure, high speed self-leaning ability, self-adaptable processing ability, arbitrary function mappingability, powerful pattern classification and pattern recognitioncapabilities formodelingare designed the complexnonlinearsystems [24]. There are many ANN models; one of them is FeedForward Neural Networks (FNN).InFNN, the processing elements(neurons) are distributed in several layers,and the structure of a 3-layer FNN is shown in Fig. 1. The intermediatelayers are known as the hidden layers, while the first and the lastlayers are known as the input and output layers, respectively. Ingeneral terms, each neuron receives signals processed and transmittedby the neurons in the preceding layer and in turn processes andtransmits them on to the next layer. The number of layers and theway in which the neurons are connected decide the architectureof the network.

The input signals ($x_1, x_2, \ldots, x_n$) are the values of the variablesrepresenting the instance of the phenomenon to be modeled. Theyare collected by the input layer which transmits them through linksto the neurons in the first hidden layer. The signals are scaled ineach link according to an adjustable parameter associated witheach connection between neurons called weight. Asis shown in Fig.1,The weight between the second neuron of input layer and the first neuron of hidden layer is w_{12}^{1} ,while the weight between the first neuron of hidden layer and the first neuron of output layer is w_{11}^{2}. Usually, the initialweight of each link is randomly set. Each neuron in the hidden layercollects the signals from the connections, adds them up andproduces an output that is a function of the sum. The mostcommonly used functions are *sigmoids*,*hyperbolic tangents* andlinear versions of the latter. The signals traverse the network fromthe input layer to the output layer, where the network response tothe inputs is collected ($y_1, y_2, \ldots, y_m$).

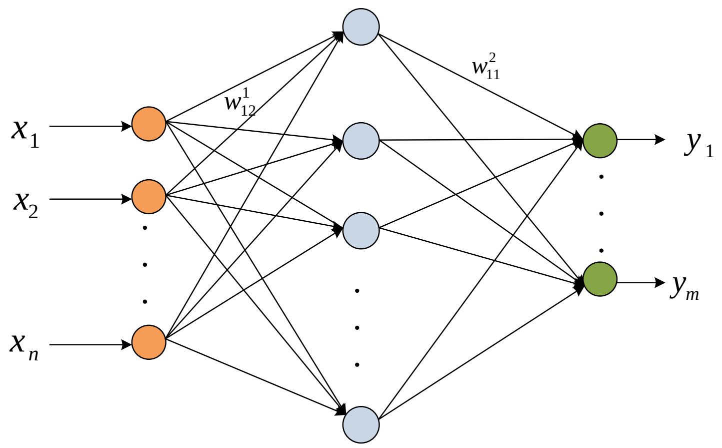

Fig.1 The structure of a 3-layer FNN

Amongst the various architectures of neural networks, theback-propagation (BP) type of FNN is the most popular and thishas been adopted in the present study. BP network used to train the networks by errorback-propagation algorithms [25].Basically, the BP algorithm works as follows: once the networkerror for a given input has been calculated, the weights of theconnections between the neurons in the last hidden layer and theoutput layer are modified according to the extent to which theseconnections have contributed to creating the error.

BP is a gradient-descent procedure which, ideally, requiresinfinitesimal changes in the connection weights.

Another popular ANN type of FNN is RadiusBased Function (RBF)neural network, which is capable of universalapproximation, utilizes basis functions in the hiddenlayer. RBFnetworks have recently attracted extensive interests in thecommunity of neural networks for a wide range of applications. They are universal approximators, possessthe best approximation property, have more compact topology than other neural networks and learn fast because of locally tuned neurons[26, 27].

3.2 Genetic algorithm

Genetic algorithm (GA) developed by Holland is based onthe Darwinian theory of biological evolution [18, 28]. It is a veryimportant stochastic search algorithm for solving optimizationproblems during the last two decades. GA has been widely andsuccessfully applied to various problems like operationresearch, image processing, and control problems[29-31], etc..GA is capable of solving wide range of complex optimization problems onlyusing three simple genetic operations (selection/reproduction, crossover, and mutation) on coded solutions(strings/chromosomes) for the parameter set, not the parameters themselves in an iterative fashion. GA considers several points in the search space simultaneously, which reduces the chance of convergence to alocal optimum. The pseudo-code algorithmdepicted in Fig. 2summarizes the general procedure of GA[28]. In the algorithm, $p(t)$ denotes a population of m individuals at generation t .GA does not have many mathematical requirements foroptimization problems. In addition, the ergodicity ofgenetic and evolution operations makes GA more effectivein the global search. Also, the easy-to-grasp implementationprocedure underlying GA provides great opportunity tohybridize them with domain-dependent heuristics towardsa more effective strategy cast in accordance with theproblem at hand. For these advantages, GA has receivedconsiderable attention for their potential of a morerobust technique.

Begin
$t \leftarrow 0$;
Initialize population $p(t)$;
Evaluate every individual in $p(t)$;
 Do
 Perform crossover in $p(t)$ to yield $p'(t)$;
 Perform mutation in $p'(t)$ to yield $p''(t)$;
 Evaluate every individual in $p''(t)$;
 Select $p(t+1)$ from $p''(t)$’
 $t \leftarrow t+1$;
 While (not termination condition)
End

Fig. 2. Genetic algorithms procedure.

Traditional GA employed binary-string encoding is not suitable for many continuousoptimization problems[32, 33]. One major drawback of a binary-string genetic algorithm isthat it encodes parameters as finite-length strings such thatmuch computation time is wasted in the encoding and decodingprocesses. Instead, the real-coding is more suitable for these types of problems. The real-coding approachseems adequate when tackling optimization problems of parameters with variables in continuous domains[34]. Instead of the codingprocesses, real-coding GA directly handles the parameters themselvesand much computation time is saved. As for the main geneticoperators, RGA is the same as binary-string GA in the reproductionprocess, but they are different in the crossover and mutationprocesses.

The application of genetic algorithm into artificial neural networkmainly includes two aspects [35]: one is to optimize theweights of network; the other is to optimize the topologicalstructure of network. This paper mainly discusses the former;the learning process of network is regarded as the process ofsearching for optimum in the weight space.

4. The hybrid intelligent optimizationmethod

In order to solve the Section 2mentioned 4 problem, we proposed a hybrid intelligent optimization methodwhich can be divided into an inner layer and an outer layer. The inner layer includes three parts. The first is the stochastic simulation and regression, whose function is to obtain the relationship $\{(a_j^i, \phi^i)\}$ between cut-off grade a_j and loss rate ϕ.The second is the BP artificialneuron networks（ANN）, whose function is to calculate $\varepsilon : \varepsilon = \varepsilon(a_t, q_t, a_j, a_r)$, i.e., the metal recovery rate as afunction of cut-off grade a_j, milling grade a_r, geographic grade a_t, and geological reserve q_t.The final part of the inner layer is the fuzzy systemused to obtain the total cost of mining C througha RBF. The outer layer is the genetic algorithm (GA). It searches for the combination of (a_j, a_r) and the weights of ANN thatmaximizes NPV while minimize the Mean SquareError (MSE) of the ANN. whereasthe inner layer carries out the local approximation,the outer layer carries out the global searching.

4.1 The relationship between cut-off grade and loss rate

Obviously, loss rate positively correlates with cut-off grade, which implies that the higher cut-off grade would result in the more abandoned ore and thereforethe higher loss rate. However, in the traditional production process, the loss rate is adopted in the calculation of mineral processing being completed, and the cut-off grade is not easy to be got.

Expression $\phi(a_j)$ cannot be directly obtained by learning without the historical record of a_j. Theoretically, to a certain extent, a_j follows normal distribution every month; ϕ and a_j appear positive relationship, namely, the bigger a_j, the bigger ϕ, vice versa. In order to express the complex relationship between a_j and ϕ, we applystochastic simulationsmethod,which hasbeen proved enlightening and useful in situations where the small volumes and moderate concentrations of reacting species make the fluctuations an important part of the system [36, 37]. This paperfirst aims at obtaining the cut-off grade based on loss rate by stochastic simulations, and then establishes the linear regression function (REG) between them.

Simulation condition: assume cut-off grade $a_j \sim N(\mu,\sigma^2)$with density function

$$f(a_j)=e^{-\frac{(a_j-\mu)^2}{2\sigma^2}} .(5)$$

The method can be describedin detail s follows:

Step1. Let $k=1$.Randomly generatetime series n interval $(\mu-\sigma,\mu+\sigma)$ with group length equals T (T represents time) .Let

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1T} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2T} \\ \cdots & \cdots & \cdots & \cdots \\ \alpha_{n1} & \alpha_{n2} & \cdots & \alpha_{nT} \end{bmatrix} \alpha_{ij} \in (\mu-\sigma,\mu+\sigma)\ i=1,2,\cdots,n ,(6)$$

denote the ranked time series from the smallest to the largest within each row. Let

$$\bar{\alpha}_j^{k} = \frac{1}{n}\sum_{i=1}^{n}\alpha_{ij} \quad (7)$$

bethe one-time simulation value of the cut-off grade a_j atthe j–thmonth ($j=1,2,\cdots,T$).

Step 2. Evaluate the cut-off grade simulated vector $\bar{\alpha}^{k}=\left(\bar{\alpha}_1^{k},\bar{\alpha}_2^{k},\cdots,\bar{\alpha}_T^{k}\right)$ .If, for a given $\varepsilon>0$, $\left|\bar{\alpha}_j^{k}-\bar{\alpha}_j^{k+1}\right|<\varepsilon$ for each $j=1,2,\cdots,T$, then the simulation is considered convergent and subsequently terminated. Otherwise increase the value of n , and repeatStep1until the subsequent Step2 terminates.

Step 3. Rank the loss rate ϕ (given data) from the smallest tothe largest in T months, match them with the simulated a_j values, and finally obtain the cut-off grade correspondingtothe loss rate in time T . The simulation flow chart is shown in Figure 1.

Step 4. Calculate the correlation coefficient between cut-off grade and loss rate. If the correlation is large (say, larger than 0.86), then regress the loss rate as a linear function (L-REG) of cut-off grade. Otherwise use other appropriate non-linear regression models.

Fig.3 displays the structure of the proposed stochasticsimulation method.

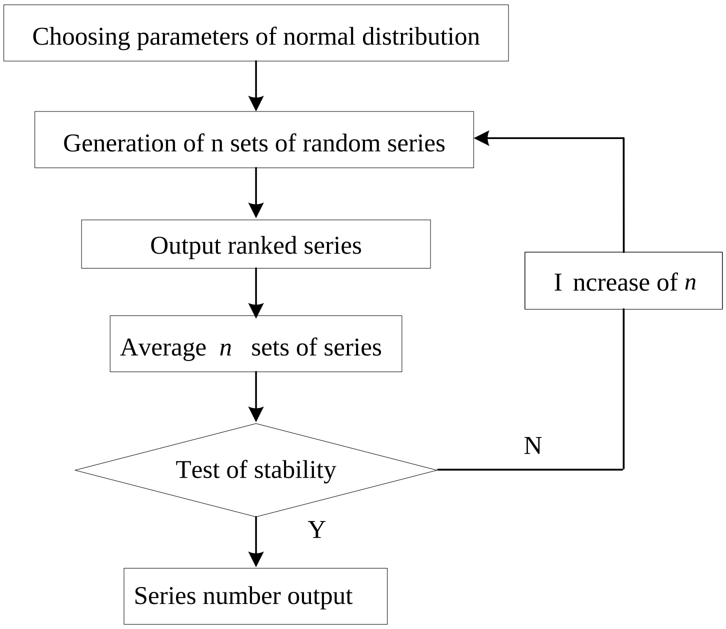

Fig.3Flow chart of cut-off grade stochastic simulation algorithm

Theorem: Under the assumption of Eq.(2), the simulated vector $\bar{\alpha}^{k}=\left(\bar{\alpha}_1^{k},\bar{\alpha}_2^{k},\cdots,\bar{\alpha}_T^{k}\right)$ converges, i.e., for a given $\varepsilon>0$,there exists a positive integer N, such that if $n>N$ then

$$\left|\bar{\alpha}_j^{k}-\bar{\alpha}_j^{k+h}\right|<\varepsilon\,,\quad j=1,2,\cdots,T\ ,(8)$$

where k, h are positive integers, $\bar{\alpha}_j^{k}$ and $\bar{\alpha}_j^{k+h}$ are two simulated values of the cut-off grade.

Proof: From Eq.(3),let $\alpha_i=(\alpha_{i1},\alpha_{i2},\cdots,\alpha_{iT})$, $i=1,2,\cdots,n$. They are independent random variables series from the normal distribution. The mathematical expectations of them are $E(\alpha_{it})=\mu$, variance is $D(\alpha_{it})=\sigma^2$. Consider random vector

$$\bar{\alpha}=\frac{1}{n}\sum_{i=1}^{n}\alpha_i=\left(\bar{\alpha}_1,\bar{\alpha}_2,\cdots,\bar{\alpha}_T\right)=\left(\frac{1}{n}\sum_{i=1}^{n}\alpha_{i1},\frac{1}{n}\sum_{i=1}^{n}\alpha_{i2},\cdots,\frac{1}{n}\sum_{i=1}^{n}\alpha_{iT}\right).(9)$$

It’s clear that

$$D\left(\frac{1}{n}\sum_{i=1}^{n}\alpha_{it}\right)=\frac{1}{n^2}\sum_{i=1}^{n}D(\alpha_{it})=\frac{1}{n^2}n\sigma^2=\frac{1}{n}\sigma^2\,,$$

and

$$E\left(\frac{1}{n}\sum_{i=1}^{n}\alpha_{it}\right)=\frac{1}{n}\sum_{i=1}^{n}E(\alpha_{it})\cdot$$

According to the famous Tchebyshev Inequality[36], for any $\varepsilon>0$,

$$P\left\{\left|\frac{1}{n}\sum_{i=1}^{n}\alpha_i-\frac{1}{n}\sum_{i=1}^{n}E(\alpha_i)\right|<\varepsilon\right\}\geq 1-\frac{\sigma^2}{n\varepsilon^2}\,.$$

Let $\varepsilon = \frac{1}{\sqrt[3]{n}}$,

$$\lim_{n\to\infty} P\left\{\left|\frac{1}{n}\sum_{i=1}^{n}\alpha_i - \frac{1}{n}\sum_{i=1}^{n}E(\alpha_i)\right| < \varepsilon\right\} = 1 .(10)$$

According to Eq.(7), $\frac{1}{n}\sum_{i=1}^{n}\alpha_i = \left(\frac{1}{n}\sum_{i=1}^{n}\alpha_{i1}, \frac{1}{n}\sum_{i=1}^{n}\alpha_{i1}, \cdots, \frac{1}{n}\sum_{i=1}^{n}\alpha_{i1}\right) = \left(\bar{\alpha}_1, \bar{\alpha}_2, \cdots, \bar{\alpha}_T\right)$ is the simulated values of cut-off grade.

4.2 The metal recovery rate computation model

The following traits exist in the mapping of cut-off grade, milling grade and the recovery rate of metal: (1) The datafrom geological and production reports are short of integrity andaccuracy; (2) In the production process, the cut-off gradeoccurs in the mining stage, milling grade occurs in mixing phase, the metal recovery rate takes place in the milling period, so there are indirectnessand backwardness; (3) The function relation is highlynonlinear and complex. Therefore, we propose to use the Back Propagation (BP) neuron network to establish models among these variablesaccording to the historical input-output data.

4.3 Calculation of total mining and milling costs

Mining and milling costs C includeboth mining cost and milling cost, which can be represented as:

$C = C(a_t, q_t, a_j, a_r)$. (8)Therefore, the mapping from 4 variables on the right side ofEq. (8) to C is not a simple mappingbut a complex one. In order to set the mapping between C, and a_t, q_t, a_j and a_r we usea RBF network which became a popular technique since the 1980s because of its simple structure, well-established theoretical basis and fast-learning speed.

4.4 Thehybrid intelligent optimization method design

4.4.1Encoding

Encode the chromosome which formed by grades (cut-off grade and milling grade) and the weights of BP network and RBF network by real-coding. The cut-off grade is g_1 and milling grade is g_2. While, $w_1, w_2, \ldots w_j$ and $w_1', w_2', \ldots w_k'$ are the weights of BP network and RBF network, respectively. The structure of chromosome is shown Fig.4.

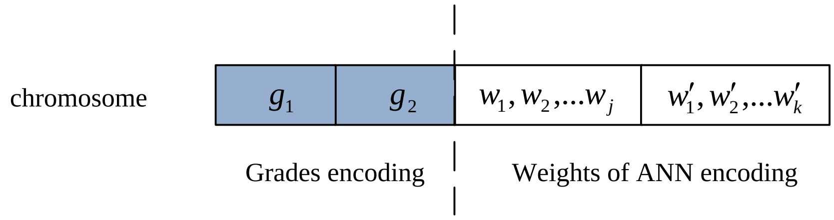

Fig.4 Diagram of chromosome

4.4.2Design of the GA operators

(1)*Selection operator*

Generally, selection operator of genetic algorithm is implementedby using roulette-wheel algorithm. The main defect of theroulette-wheel algorithm is that local optimal gene dominates thewhole generation, while global optimum would likely be eliminatedbefore emerging. Therefore, the roulette wheel with elitistselection method is considered as the selection mechanism in thisproposed real-coding GA algorithm[38]. This method including two phases,at first, the elitist strategy was adopted. The generation whose fitnessis within top 10% will be directly copied to the new generation. Then the rest of chromosome is selected by roulette wheel.

(2) *Crossover operator*

Crossover is a mechanism of randomly exchanging informationbetween two chromosomes. Thecrossovers mechanisms forbinary and real coding are different. In this study, a crossover ofreal-coding follows: Let $chrom1$ and $chrom1$ are two chromosomes, r_1 and r_2 are two independently distributedrandom variables with range [0, 1].We can get two new chromosomesby Eq. (9).

$$\begin{cases} chrom1' = chrom1 + (1 - r_1) \cdot (chrom2 - chrom1) \\ chrom2' = chrom2 + (1 - r2) \cdot (chrom1 - chrom2) \end{cases} \tag{9}$$

(3) *Mutation operator*

The purpose of mutation operation is to make genetic algorithmobtain local random research capability through varying certaingenes of chromosome. If a chromosome is selected for mutation,one gene is randomly selected for changing their values. In the respectof real representations the process is particularly simple:let p is a parent,and then the child chromosome is $p' = p + \varepsilon$,where ε a distributed random variables with range [0, 1] is.

(4) *Adaptive probability of crossover and mutation*

Traditional crossover and mutation operators are based on arandomization mechanism, i.e., generating a cut point, and determiningthe position of the bit shifted by mutation of the solution.

But this is not the case in natural evaluation which is mimicked bythe GA. Actually renewing the bits of the solution is dynamic oradaptive, but not random. The slightly modified adaptive probabilitiesof crossover and mutation given by Srinivas and Patnaikare used in this study to choose the probability of mutationand crossover according to the fitness value of the solutions[39]. Themodified expression of p_c and p_m are as follows:

$$Pc=\begin{cases}(f_{\max}-f')/f_{\max}-f_{ave} & if \quad f'\geq f_{ave}\\ 1 & if \quad f'<f_{ave}\end{cases} \tag{10}$$

$$Pm=\begin{cases}0.5\cdot(f_{\max}-f')/f_{\max}-f_{ave} & if \quad f'\geq f_{ave}\\ (f_{ave}-f)/(f_{ave}-f_{\min}) & if \quad f<f_{ave}\end{cases} \tag{11}$$

Here, f is the fitness of an individual, f_{ave} is the average fitness value ofthe population, and $f_{\max}$ and $f_{\min}$ are the maximum and minimumfitness value of the population respectively. f' is the larger of thefitness values of the solutions to be crossed.

4.4.3Fitness function

A fitness value is given by Eq. (12).

$$fitness=\frac{1}{E_{BP}+E_{RBF}}+NPV \tag{12}$$

Where E_{BP} and E_{RBF} are the mean squared error of BP networks and RBF networks, respectively. *NPV* is the total net present value as in Eq.(1).

4.5The steps of hybrid intelligent grades optimization

We integratethe proposed genetic algorithm with stochastic simulations and regression, neuron networks to optimizecut-off grade a_j and milling a_r so that NPV can be maximized and minimize the MSE of the ANN. The implementation of the entirealgorithm is shown as follows:

(1) Normalize raw data.

(2) Initialize correlation parameters of the hybrid optimization algorithm:the size of population pop_size, the max generation $\max gen$; initial parameter k_1,k_2,k_3,k_4 of adaptive p_c and p_m; the max learn epochs of BP network EP_{BP} and RBF network EP_{RBF}.

(3) Generate initial real-coding population chromosome $[g_1,g_2,W,W']$ randomly. g_1 and g_2 arethe code of cut-off grade and milling grade, while W and W' are the weights of BP network and RBF network, respectively.

(4)Input the individual to stochastic simulation (SS)and the linear regression function (REG) ,get the relationship between a_j and ϕ.

(5) According to chromosomes, set a BP network which uses grade value a_j, namely g_1; a_r ,namely g_2, recoverable reserves q_t, and geographical grade a_t as the inputs while the metal recovery rate ε as the output. Meanwhile, the BP network initial weights are given by W which is a part of the chromosome.

Train the network by BP algorithm for EP_{BP} epochs.

(6) Set a RBF network which uses g_1, g_2 ,recoverable reserves, geographical grade as the inputs while the total cost C as the output. The RBF network initialweights are also given by W' and then train the network by BP algorithm for EP_{RBF} epochs.

(7) Calculate the fitness function according Eq.(12).

(8) Carry out the genetic operation (selection, crossover, and mutation) according to fitness function to obtain new generation of population.

(9) Repeat step (4) to step(7) until the termination criterion is met. Finally output the optimized grade values.

The flow chart of the algorithm is given in Fig.5.

5. Case study

Daye iron mine is the main domestic supply base of the Wuhan steel andiron (Group) Corp., which is a very famous large enterprise inChina. Daye iron mine locates at the Tieshan county in the city ofHuangshi, Hubei Province, about 90 km far from Wuhan city, withthe national highway No 206 passing through the mine. Daye ironmine has six big ore deposit from west to east, which are dividedin two mining workshops. Now it faces the following problems.First, the cut-off grade scheme was established according to themining technique, milling technology and

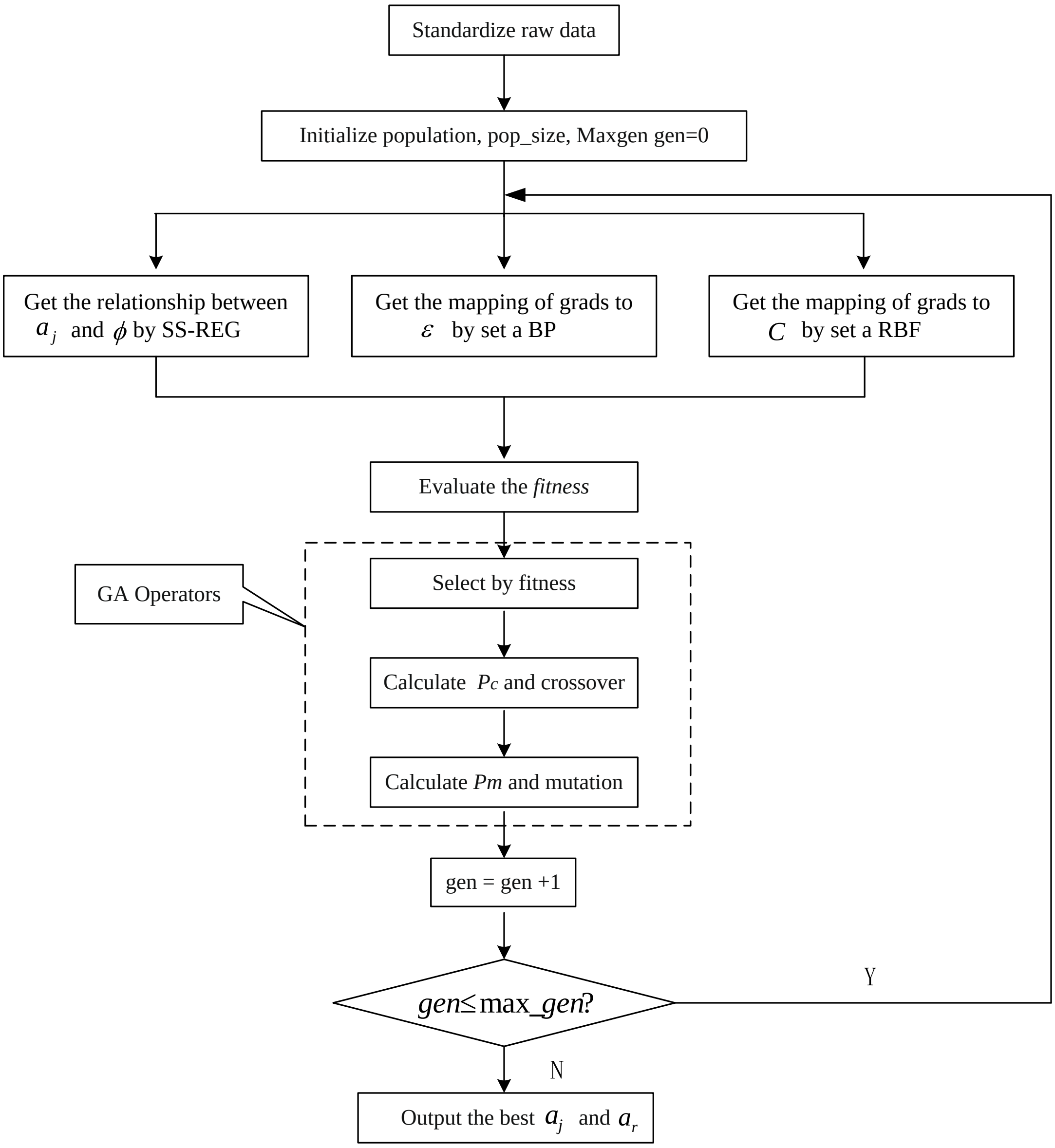

Fig.5Flow chart of hybrid intelligent system algorithm

price of the concentratein 1990s, but now the fact that the scheme is reasonable or not needs to bestudied. Second, the geological condition, mining and milling technologyhave been greatly changed Along with reconstruction of millingprocess, it is necessary to find out the optimum grade of crude ore.Therefore, with the improvement in iron mining technology and the upgrade of ore selection process, there is a great demand to optimize cut-off grade and pre-selection grade of crude ore to guide the mining and selection process. Table 1 presents the relative raw data of Daye mine of 35 months from January 2005 to November 2007.

Table 1 The relative raw data of Daye iron mine

Time	recoverable reserves (ton)	Geological grade (%)	Cut-off grade (%)	Milling grade (%)	Metal recovery rate (%)	Loss rate (%)	Total cost (ten thousand Yuan)

200501	87400	52.73	17.64	45.63	88.36	17.17	3241.85
200502	79758	52.2	16.73	43	72.99	15.76	1963.25
200503	112487	52.59	16.88	46.31	70.50	15.84	2629.59
200504	117389	52.98	15.89	44.77	70	14.68	3180.41
200505	117081	52.79	16.31	41.64	70.35	15.13	3554.68
200506	118176	52.89	16.55	44.37	77.5	15.53	2886.44
200507	116850	52.88	17.21	44.86	77.92	16.12	2916.28
200508	124078	52.99	17.11	45.67	79.37	16.02	3315.72
200509	119443	52.77	17.48	44.15	76	16.52	2896.86
200510	129305	53.33	17.31	45.11	72.13	16.45	3354.84
200511	104938	53.58	17.71	45.63	84.86	17.30	2875.79
200512	62308	54	17.00	42.22	51.72	15.88	5631.27
200601	115203	53.23	17.39	42.42	75.18	16.48	2839.24
200602	117173	52.95	18.07	42.59	70.46	18.42	2879.86
200603	111742	52.74	18.22	42.61	77.8	18.62	3342.96
200604	133607	52.71	18.52	41.93	67.69	19.02	3716.11
200605	142315	52.74	18.89	43.01	71.56	19.75	3784.27
200606	146743	52.78	17.85	42.72	56.97	18.23	4039.98
200607	129853	52.88	18.44	42.64	79.57	18.81	3964.99
200608	132260	52.81	17.93	42.36	78.06	18.35	4447.65
200609	125422	52.3	17.56	43.99	77.67	17.16	4384.32
200610	80204	52.52	17.78	42.04	66.96	17.95	4063.94
200611	124660	53.14	18.29	42.42	80.5	18.64	4564.21
200612	124799	53.19	18.14	40.72	76.95	18.51	3996.54
200701	112212	52.84	18.07	42.20	78.49	18.42	3303.39
200702	143936	52.78	18.36	41.84	77.67	18.67	2840.79
200703	144099	52.95	18.60	41.37	77.74	19.19	3701.04
200704	135334	52.87	18.69	41.56	68.41	19.54	3951.00
200705	146158	53.25	18.79	43.31	83.06	19.63	3656.89
200706	149207	53.13	19.00	43.05	77.14	20.00	4602.53
200707	149160	53.18	20.10	42.57	76.77	20.38	4757.97
200708	149207	52.81	19.12	41.52	66.77	20.10	4393.16
200709	149160	52.87	19.69	41.45	69.05	20.25	4039.37
200710	146189	52.59	19.45	41.75	79.2	20.21	4279.91
200711	143134	53.05	19.27	41.05	73.27	20.16	5285.25

5.1 Calculation of loss rate

The cut-off grade in Daye mine varied around 18%. Assume that cut-off grade a_j (in percentage) follows a normal distribution with mean $\mu = 18$ and $\sigma = 1$.The density functionis

$$f(a_j) = e^{-\frac{(a_j - 18)^2}{2}}. \tag{10}$$

According to the method introduced in 2.1, we obtain corresponding cut-off grade (column4) from simulations.

The correlation coefficient between the cut-off grade and loss rate is calculated as 0.97419, with an estimatedregression function of loss rate ϕ as $\phi = 1.7a_j - 12$ (see Fig.6).

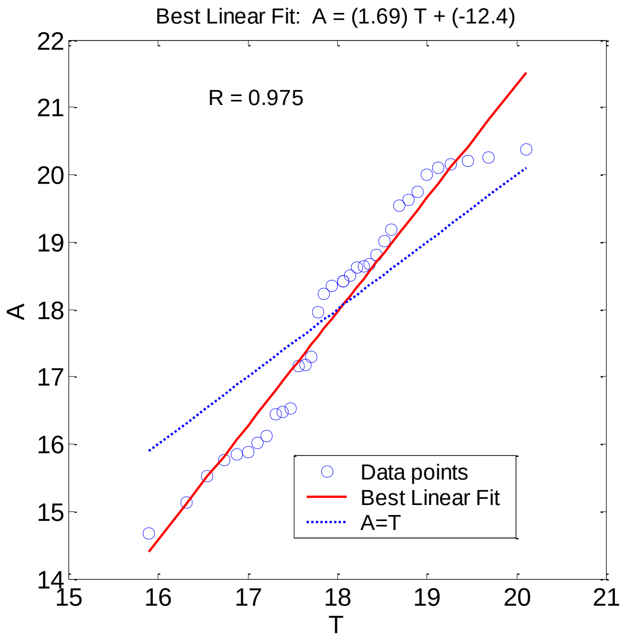

Fig.6Regression fitting

5.2 Calculationmetal recovery rate

Additional data are obtained from the geographical reports, production reports and cost reports of Daye mine from January 2005 to November 2007. These data are presentedin Table1. We use these data from January 2005 to September 2007 as training data and those datafrom October 2007 to November 2007 as prediction data to establish a mapping from four factors to metal recovery rateby BP network.

Use $newff()$ function in matlab6.5 to establish BP neural network, which contains 4 inputnodes, 1 hidden layer, and 1output node. The transfer functions of hidden and outputlayer are *tansig*and *purelin*, respectively. The initial weights of network are given by W of each individual. The epochs of learning by BP algorithm are 500. The hidden nodeshave been chosen from 3 to 7. Table 2 shows that the best hidden nodes are 5.

Table2 BP networks results comparison with different hidenods

Hidden nodes	Training data MSE	Testing data MSE
3	0.01623	0.02530
4	0.01011	0.01852

5*	0.00786	0.01650
6	0.00360	0.09840
7	0.00015	0.89512

5.3 Calculation of cost

We still use data from January 2005 to September 9 2007 as training data and from October 2007 to November 2007 as prediction data (see Table 2). We use RBF network to establish a mapping of recoverable reserves q_t, geographical grade a_t, cut-off grade a_j, millinggrade a_r, to total cost C. The sample simulation graph is shown in Fig.8. Again the graph suggests that the neuron network has good simulation estimation ability.

Use *newrb*() function in matlab6.5 to establish RBF neural network, which contains a 4 inputnodesand 1output node. The initial weights of network are given by W' of each individual. And the hidden nodes are automaticallydetermined by *newrb*() function.Theepochs of learning are 300.

5.4 Results

We chose the range of cut-off grade as 15.8-20.1%, the range of milling as 40.7-46.4%. The fitness function is give by Eq. (12) with β =64% , p =517.12 , i =0.05. Population $pop\_size = 150$; $\max\_gen = 500; k_1 = k_2 = 0.3; k_3 = k_4 = 0.1$. The total length of the chromosome is 42 which the grade is 2, weight of BP is 25 and RBF is 15. Make programsin Matlab 6.5. When the training reaches the $\max\_gen$,the best fitness value is 194.3054.Whilethe E_{BP} is 0.00786 and E_{RBF} is 0.00468.; the NPV is 114.5654 million.

The metal recovery rate and total cost simulated and predicted results are shown in Fig.7 and Fig.8, respectively. The 34 and 35 sample index is the prediction sample. We can see that the ANN composed in this study has a better simulation and prediction.

According to Eq. (1), the calculation results are shown in the second row of Table3.

Table 3Comparisons of NPV: before and after optimization

	Cut-off grade (%)	Milling grade (%)	Loss rate (%)	Recovery (%)	NPV (million)
after optimization	17. 8337～17. 836 7	46.4	26.86	73.14	114.5654
Before optimization	18	41~43	30.6	69.4	107.8676

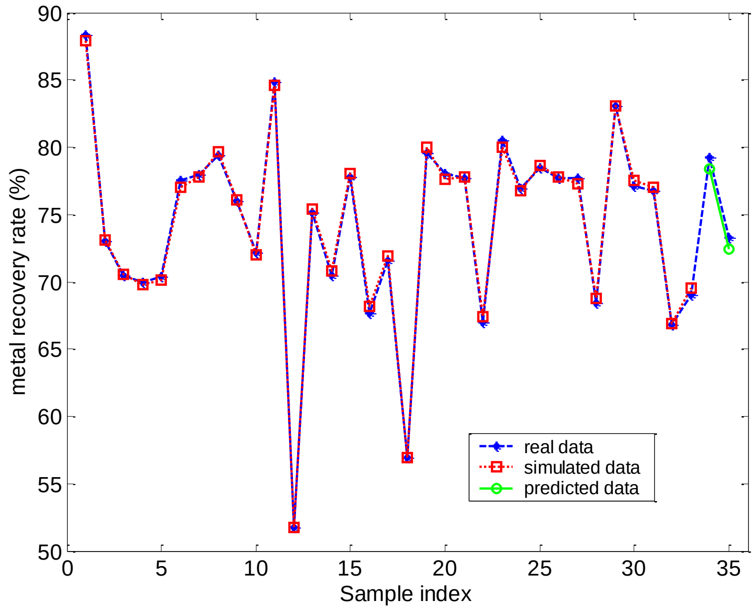

Fig.7 Metal recovery rate simulation

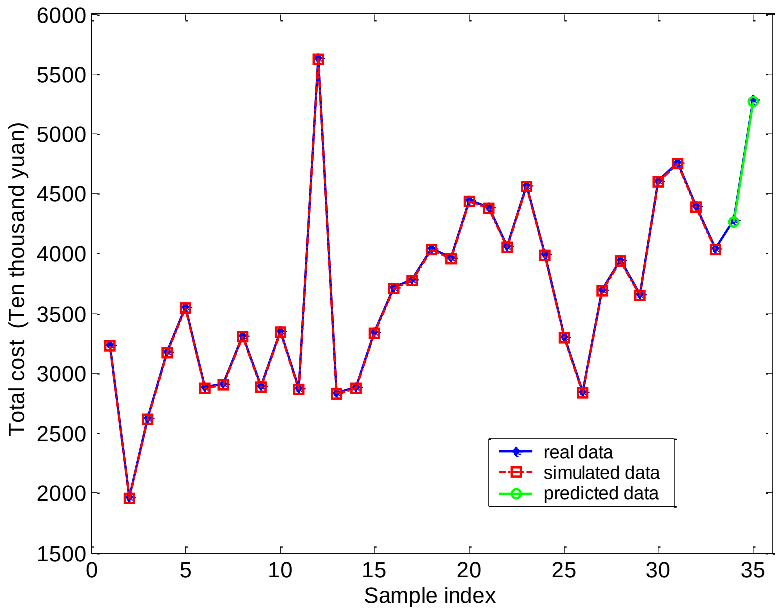

Fig.8 Cost simulation.

Table2 suggests that if we had used the optimal cut-off grade of 17.8337~17.8367%and the optimal milling grade of 46.4% in production, Daye minewould achieve a NPV of 114.5654 milling Yuan from 01/2007 to 12/2007 while the real NPV is 107.8676milling under thecurrent grades as 18% and 41~43%.There is 6. 6978 milling Yuan added compare to current grades.

6. Conclusion

This research is based on an integrated intelligent system from the genetic algorithm, random simulationsand neuron networks. The relationship among the cost, revenue, grade, and metal recovery rate is highly complex and non-linear. The aim of this research is to obtain the optimized combination of cut-off grade and millinggrade to maximizethe profit of production process. The creativity of this research includes:

1. We established a model to optimize cost, revenue, grade, and metal recovery rate in production process. This model includes 3 unknown functions related with cut-off grade and millinggrade.

2. We designed a simple random simulation technique to obtain the cut-off grade related with the loss rate.

3. We efficiently integrated some soft computing methods through a neural network. The inner layer of the network carries out the local approximation while the outer layer carries the out global search. The integration of these two layers can avoid the common problem of the local minimization of neural network.

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