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Efficiency evaluation of multistage supply chain with data envelopment analysis models

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Abstract: In order to evaluate the multistage supply chain efficiency, an appropriate performance evaluation system is importantly required. In practice, a representative multistage supply chain has three members composing a supplier-manufacturer-retailer structure, and has intermediate measures connecting these three supply chain members. Existing Data Envelopment Analysis (DEA) models have difficulties in measuring these kinds of supply chain efficiencies directly. In this paper, we develop several DEA based models for characterizing and measuring these multistage supply chain efficiencies with the consideration of the intermediate measures. We illustrate the models in a three-stage supply chain context which can represent different relationships between the supplier, manufacturer and retailer when they are treated as in different supply chain structures: i) the non-cooperative, ii) the partial-cooperative, and iii) the cooperative supply chain structure. Moreover, the general DEA frameworks for multistage supply chain models are proposed and these models are demonstrated with an illustrative example.
Keywords: Data envelopment analysis; Efficiency evaluation; Multistage supply chain

1. Introduction

Supply chain performance evaluation is a basic work for an organization to promote its supply chain efficiency. In order to evaluate the multistage supply chain efficiency, an appropriate performance evaluation system is importantly required. One difficulty presented in evaluating the performance of a multistage supply chain and its members is the existence of multiple measures that characterize the performance of chain members. Besides, the other difficulty is the existence of conflicts between the supply chain members with regard to specific measures.

Data envelopment analysis (DEA), first proposed by Charnes, Cooper and Rhodes (1978), is a linear programming based methodology for evaluating the relative efficiency of each member of a set of organizational units, which are called decision making units (DMUs). The DMUs consume various specified inputs to produce various specified outputs. DEA evaluates the efficiency of each DMU relative to an empirical production possibility frontier which is determined by all DMUs under appropriate assumptions with respect to the returns to scale and the orientation (Banker, Charnes and Cooper, 1984). The traditional standard DEA model does not consider the internal structure of the DMUs, i.e., it treats each DMU as a black box by considering only the input consumed and the output produced by each DMU. This DEA approach has difficulties in measuring supply chain efficiencies, which include the supply chain's efficiency and it members' efficiencies, because it neither provides insight of the internal operations of each DMU nor makes locations of efficiency or inefficiency to the multiple sub-stages in each DMU(Reiner and Hofmann, 2006; Xu, Li and Wu, 2009). Thus, the standard DEA approach should be improved to emphasize the stages of the supply chain production process, i.e., the complicated series supply chain production process should be divided into sub process, in which the intermediate products are considered. Especially some of the intermediate products are outputs from a sub-process on the one hand and are inputs to another sub-process on the other hand.

The researches of Seiford and Zhu (1999), Zhu (2000), Fare and Grosskopf (2000), and Sexton and Lewis (2003), are some examples of this approach. In their papers, the complicated production process is composed of two sub processes connected in series. Seiford and Zhu (1999) consider a production process of commercial bank as two stages of profitability and marketability. The inputs of the first stage denote the inputs of the bank production process, and the outputs of the second stage denote the outputs of the bank production process. In addition, there are intermediate products which are the outputs of the first stage as well as the inputs of the second stage. The efficiencies of the first stage, second stage, and the bank whole production process are calculated through three independent DEA models. With the similar idea, Zhu (2000) analyzes the efficiencies of the 500 fortune companies. Fare and Grosskopf (2000) develop a network DEA approach to model general multiple stage processes with intermediate inputs and outputs. Their network model has a general structure which allows to be applied to a variety of situations include the intermediate products, the allocation of fixed factors, and certain dynamic systems. Their dynamic network model is generally consisted of two distinct sub technologies $P(t)$ and $P(t+1)$, one for each evaluation period. The outputs from $P(t)$ are divided into two parts: normal outputs which are the consists of the final outputs of the whole system, and intermediate outputs which are used as inputs at $P(t+1)$. The inputs to $P(t+1)$ are also divided into two parts: intermediate inputs which are same to the intermediate outputs from $P(t)$, and normal inputs which directly come from the outside of the system. Sexton and Lewis (2003) show in their paper how to use DEA approach to model two stage DMUs and apply their model to major league baseball. They demonstrate the model's advantages over standard one stage DEA as: i) their model can detect the inefficiency that may be undetected by the one stage DEA model; ii) their model can provide greater managerial insight into the locations of inefficiency within the whole system.

The two stage sub production process concept has also been applied to evaluate the performance of information technology (Chen and Zhu, 2004; Chen, Liang, Yang and Zhu, 2006a), insurance company (Kao and Hwang, 2008; Chen, Cook, Li and Zhu, 2009a; Chen, Liang and Zhu, 2009b), and supply chain (Liang, Yang, Cook and Zhu, 2006; Chen, Liang and Yang, 2006b). Chen and Zhu (2004) proposed a DEA framework which considers a two stage process as efficient when each stage is efficient. In the two stage DEA model of Kao and Hwang (2008), the efficiency of the overall process is the product of the efficiencies of the two stages, and the multipliers on the intermediate measures are the same for the two stages. The assumption of same multipliers also links the two stages, because if one assumes that the multipliers on the intermediate measures are not the same, then the two stage DEA model is equivalent to treating each stage independently of the other stage by using the standard DEA model. Chen Cook, Li and Zhu (2009a) point out the limitation of the Kao and Hwang's model, and develop an additive two stage DEA approach in which the overall efficiency is expressed as a weighted sum of the efficiencies of the individual stages. Chen et al. (2009b) also prove the equivalence between the Kao and Hwang (2008) model and the Chen and Zhu (2004) model. Liang, Yang, Cook and Zhu (2006) develop several DEA based approaches for characterizing and evaluating supply chain and its member efficiencies, when the intermediate measures are considered in the performance evaluation. The two stages of the supply chain in their models are illustrated as a seller-buyer structure, and the relationship between them is treated as a leader-follower and a cooperative relationship. With the same model structure, Chen, Liang and Yang (2006b) propose a DEA game model approach for supply chain efficiency evaluation. Other application of the two stage DEA concept can be found in the research of performance evaluation of economics of OECD countries (Prieto and Zofio, 2007), and manufacturing industrial (Liu and Wang, 2009).

In our current research, we consider a representative multistage supply chain which has three members composing a supplier-manufacturer-retailer structure, and has intermediate measures connecting these supply chain members. Expending the idea for two stage DMU efficiency measuring in Liang, Yang, Cook and Zhu (2006), and considering not only intermediate inputs and outputs but normal inputs and outputs in Fare and Grosskopf (2000), we develop several DEA based models for characterizing and measuring these multistage supply chain efficiencies. And we illustrate the models in a three stage supply chain context which can represent different relationships between the supplier, manufacturer and retailer when they are treated as different chain structures: i) the non-cooperative supply chain structure; ii) the partial-cooperative supply chain structure; and iii) the cooperative supply chain structure. In the non-cooperative structure, the supplier is first evaluated, the manufacturer is second evaluated using the information related to supplier's efficiency, and the retailer is third evaluated using both of the supplier's and manufacturer's information about efficiencies. We name this structure backward non-cooperative structure. The evaluation order also can be reversed, and we call it forward non-cooperative structure. In the partial-cooperative structure, an alliance is constructed between supplier and manufacturer, or manufacturer and retailer. The alliance is first evaluated, and then with the alliance's information about efficiency which is defined as the arithmetic mean of the alliance members' efficiencies, the supplier or retailer is evaluated. And in the cooperative structure, all the supply chain members are evaluated simultaneously using a joint efficiency which is defined as the arithmetic mean of the supplier's, manufacturer's and retailer's efficiencies.

It is important to emphasize that the primary work of our study is to provide an analytical framework within which to measure the efficiency of multistage supply chain. While it is the case that at present in many supply chains, data may not be complete enough to permit one to conduct such models, the models do serve several important purposes. Firstly, these models provide a methodology for performing 'what if' analyses on a general multistage supply chain and different supply chain structures are assumed to simulate their operations. Secondly, in those specific supply chains where relevant evaluation data are complete, these models will play a useful role. Furthermore, these models can provide important insights into supply chain operations since the managers may have strong motivations to identify their supply chain performance and work toward a more competitive supply chain structure for managing their supply networks.

The structure of the paper is organized as follows. In section 2, 3 and 4, we develop models for the non-cooperative, partial-cooperative and cooperative supply chain structures respectively. In section 5, the general frameworks for multistage supply chain DEA models are presented. These models are then demonstrated with an illustrative example in Section 6. Section 7 concludes the paper.

2. The non-cooperative multistage supply chain models

The multistage supply chain is described in Figure 1, in which the supplier, manufacturer and retailer are denoted by A, B and C, respectively. The whole supply chain is considered as a DMU, where X_A, X_B and X_C are the normal input vectors (e.g. labor, operation cost, fixed cost, shipping cost, material, etc.) of A, B and C respectively, and Y_{A2}, Y_{B2}, and Y_C are the normal output vectors (e.g. number of final products shipped, sales, profit, etc.) of A, B and C, respectively. Y_{A1} is the output vector of A and also the input vector of B, then Y_{B1} is the output vector of B and also the input vector of C. Therefore, Y_{A1} and Y_{B1} (e.g. number of various kinds of intermediate products shipped) are considered as the intermediate measures which link the supply chain members.

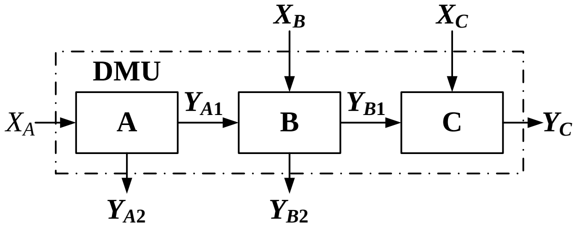

Figure 1. Three stage supply chain structure

Suppose there are n homogenous supply chains (DMUs) as above. The CCR DEA model (Charnes, Cooper and Rhodes, 1978) for the supply chain overall efficiency measure is the following model (1).

$$\max \frac{\sum_r u_r^{A2} y_{rj_0}^{A2} + \sum_r u_r^{B2} y_{rj_0}^{B2} + \sum_r u_r^{C} y_{rj_0}^{C}}{\sum_i v_i^{A} x_{ij_0}^{A} + \sum_i v_i^{B} x_{ij_0}^{B} + \sum_i v_i^{C} x_{ij_0}^{C}}$$

$$s.t. \quad \frac{\sum_r u_r^{A2} y_{rj}^{A2} + \sum_r u_r^{B2} y_{rj}^{B2} + \sum_r u_r^{C} y_{rj}^{C}}{\sum_i v_i^{A} x_{ij}^{A} + \sum_i v_i^{B} x_{ij}^{B} + \sum_i v_i^{C} x_{ij}^{C}} \le 1, \tag{1}$$

$$u_r^{A2}, v_i^{A}, u_r^{B2}, v_i^{B}, u_r^{C}, v_i^{C} \ge 0, \; j = 1, 2, \ldots, n.$$

Model (1) only considers the inputs and outputs of the whole supply chain, but misses the intermediate measures which link the supply chain members. Model (1) also can not indicate the individual efficiency of each supply chain member. Therefore, it is considered as a black box model.

In order to evaluate the supply chain performance as well as its members' performance, and consider the relationship between the supplier, manufacturer and retailer. We first propose the non-cooperative models, in which the supplier is first evaluated, the manufacturer is second evaluated using the information related to supplier's efficiency, and the retailer is third evaluated using both of the supplier's and manufacturer's information of efficiencies.

Firstly, the efficiency of the supplier is measured in model (2), which can be easily translated into the standard DEA multiplier model.

$$\max \frac{\sum_r u_r^{A1} y_{rj_0}^{A1} + \sum_r u_r^{A2} y_{rj_0}^{A2}}{\sum_i v_i^{A} x_{ij_0}^{A}} = \theta_A$$

$$s.t. \quad \frac{\sum_r u_r^{A1} y_{rj}^{A1} + \sum_r u_r^{A2} y_{rj}^{A2}}{\sum_i v_i^{A} x_{ij}^{A}} \le 1, \tag{2}$$

$$u_r^{A1}, u_r^{A2}, v_i^{A} \ge 0, \; j = 1, 2, \ldots, n.$$

The optimal solution of model (2) are u_r^{A1*}, u_r^{A2*}, and v_i^{A*}, and the optimal value θ_A^* is the supplier's efficiency. Then, the efficiency of the manufacturer is measured in model (3).

$$\max \frac{\sum_r u_r^{B1} y_{rj_0}^{B1} + \sum_r u_r^{B2} y_{rj_0}^{B2}}{\sum_i v_i^{B} x_{ij_0}^{B} + \sum_r u_r^{A1} y_{rj_0}^{A1}} = \theta_{AB}$$

$$s.t. \quad \frac{\sum_r u_r^{B1} y_{rj}^{B1} + \sum_r u_r^{B2} y_{rj}^{B2}}{\sum_i v_i^{B} x_{ij}^{B} + \sum_r u_r^{A1} y_{rj}^{A1}} \le 1,$$

$$\frac{\sum_r u_r^{A1} y_{rj_0}^{A1} + \sum_r u_r^{A2} y_{rj_0}^{A2}}{\sum_i v_i^{A} x_{ij_0}^{A}} = \theta_A^*, \tag{3}$$

$$\frac{\sum_r u_r^{A1} y_{rj}^{A1} + \sum_r u_r^{A2} y_{rj}^{A2}}{\sum_i v_i^{A} x_{ij}^{A}} \le 1,$$

$$u_r^{A1}, u_r^{A2}, v_i^{A}, u_r^{B1}, u_r^{B2}, v_i^{B} \ge 0, \; j = 1, 2, \ldots, n.$$

In model (3), the manufacturer's efficiency is measured under the condition that the supplier's efficiency remains at θ_A^*. The optimal solution of model (3) are $u_r^{A1*}, u_r^{A2*}, v_i^{A*}, u_r^{B1*}, u_r^{B2*}$, and v_i^{B*}, and the optimal value θ_{AB}^* is the manufacturer's efficiency when the supplier has first achieved its best performance. Model (3) is equivalent to the following standard DEA multiplier Model (4) according to Charnes and Cooper transformation (Charnes and Cooper, 1962):

$$
\begin{aligned}
\max & \sum\nolimits_r \mu_r^{B1} y_{rj_0}^{B1} + \sum\nolimits_r \mu_r^{B2} y_{rj_0}^{B2} = \theta_{AB} \\
s.t. \quad & \sum\nolimits_r \mu_r^{B1} y_{rj}^{B1} + \sum\nolimits_r \mu_r^{B2} y_{rj}^{B2} - \sum\nolimits_i \omega_i^B x_{ij}^B - \sum\nolimits_r \mu_r^{A1} y_{ij}^{A1} \le 0, \\
& \sum\nolimits_i \omega_i^B x_{ij_0}^B + \sum\nolimits_r \mu_r^{A1} y_{ij_0}^{A1} = 1, \\
& \sum\nolimits_r \dot{\mu}_r^{A1} y_{ij_0}^{A1} + \sum\nolimits_r \mu_r^{A2} y_{ij_0}^{A2} = \theta_A^*, \\
& \sum\nolimits_r \dot{\mu}_r^{A1} y_{ij}^{A1} + \sum\nolimits_r \mu_r^{A2} y_{ij}^{A2} - \sum\nolimits_i \omega_i^A x_{ij}^A \le 0, \\
& \sum\nolimits_i \omega_i^A x_{ij_0}^A = 1, \\
& \mu_r^{A1}, \dot{\mu}_r^{A1}, \mu_r^{A2}, \omega_i^A, \mu_r^{B1}, \mu_r^{B2}, \omega_i^B \ge 0,\ j = 1, 2, ..., n.
\end{aligned} \tag{4}
$$

At last, the efficiency of the retailer is measured in model (5).

$$
\begin{aligned}
\max & \frac{\sum_r u_r^C y_{rj_0}^C}{\sum_i v_i^C x_{ij_0}^C + \sum_r u_r^{B1} y_{rj_0}^{B1}} = \theta_{ABC} \\
s.t. \quad & \frac{\sum_r u_r^C y_{rj}^C}{\sum_i v_i^C x_{ij}^C + \sum_r u_r^{B1} y_{rj}^{B1}} \le 1, \\
& \frac{\sum_r u_r^{B1} y_{rj_0}^{B1} + \sum_r u_r^{B2} y_{rj_0}^{B2}}{\sum_i v_i^B x_{ij_0}^B + \sum_r u_r^{A1} y_{rj_0}^{A1}} = \theta_{AB}^*, \\
& \frac{\sum_r u_r^{B1} y_{rj}^{B1} + \sum_r u_r^{B2} y_{rj}^{B2}}{\sum_i v_i^B x_{ij}^B + \sum_r u_r^{A1} y_{rj}^{A1}} \le 1, \\
& \frac{\sum_r u_r^{A1} y_{rj_0}^{A1} + \sum_r u_r^{A2} y_{rj_0}^{A2}}{\sum_i v_i^A x_{ij_0}^A} = \theta_A^*, \\
& \frac{\sum_r u_r^{A1} y_{rj}^{A1} + \sum_r u_r^{A2} y_{rj}^{A2}}{\sum_i v_i^A x_{ij}^A} \le 1, \\
& u_r^{A1}, u_r^{A2}, v_i^A, u_r^{B1}, u_r^{B2}, v_i^B, u_r^C, v_i^C \ge 0,\ j = 1, 2, ..., n.
\end{aligned} \tag{5}
$$

Under the condition that the supplier has first achieved its best performance, and the manufacturer's efficiency remains at θ_{AB}^*, model (5) gives the retailer's efficiency θ_{ABC}^* as the optimal value, and the optimal solution as $u_r^{A1*}, u_r^{A2*}, v_i^{A*}, u_r^{B1*}, u_r^{B2*}, v_i^{B*}, u_r^{C*}$, and v_i^{C*}. Model (5) is equivalent to the following standard DEA multiplier Model (6) according to Charnes and Cooper transformation (Charnes and Cooper, 1962):

$$
\begin{aligned}
&\max \sum\nolimits_r \mu_r^C y_{rj_0}^C = \theta_{ABC} \\
&s.t.\quad \sum\nolimits_r \mu_r^C y_{rj}^C - \sum\nolimits_i \omega_i^C x_{ij}^C - \sum\nolimits_r \mu_r^{B1} y_{rj}^{B1} \le 0, \\
&\qquad \sum\nolimits_i \omega_i^C x_{ij_0}^C + \sum\nolimits_r \mu_r^{B1} y_{rj_0}^{B1} = 1, \\
&\qquad \sum\nolimits_r \dot{\mu}_r^{B1} y_{rj_0}^{B1} + \sum\nolimits_r \mu_r^{B2} y_{rj_0}^{B2} = \theta_{AB}^*, \\
&\qquad \sum\nolimits_r \dot{\mu}_r^{B1} y_{rj}^{B1} + \sum\nolimits_r \mu_r^{B2} y_{rj}^{B2} - \sum\nolimits_i \omega_i^B x_{ij}^B - \sum\nolimits_r \mu_r^{A1} y_{rj}^{A1} \le 0, \\
&\qquad \sum\nolimits_i \omega_i^B x_{ij_0}^B + \sum\nolimits_r \mu_r^{A1} y_{rj_0}^{A1} = 1, \\
&\qquad \sum\nolimits_r \dot{\mu}_r^{A1} y_{rj_0}^{A1} + \sum\nolimits_r \mu_r^{A2} y_{rj_0}^{A2} = \theta_A^* \times \sum\nolimits_i \omega_i^A x_{ij_0}^A, \\
&\qquad \sum\nolimits_r \dot{\mu}_r^{A1} y_{rj}^{A1} + \sum\nolimits_r \mu_r^{A2} y_{rj}^{A2} - \sum\nolimits_i \omega_i^A x_{ij}^A \le 0, \\
&\qquad \sum\nolimits_i \omega_i^A x_{ij_0}^A = 1, \\
&\qquad \mu_r^{A1}, \dot{\mu}_r^{A1}, \mu_r^{A2}, \omega_i^A, \mu_r^{B1}, \dot{\mu}_r^{B1}, \mu_r^{B2}, \omega_i^B, \mu_r^C, \omega_i^C \ge 0,\ j = 1, 2, \ldots, n.
\end{aligned} \tag{6}
$$

Then, the efficiency of the whole supply chain can be defined as the arithmetic mean of the supply chain members' efficiencies in (7).

$$
E_{ABC} = w_1\theta_A^* + w_2\theta_{AB}^* + w_3\theta_{ABC}^*, \text{ and } \ w_1 + w_2 + w_3 = 1. \tag{7}
$$

where w_1, w_2 and w_3 are decision maker specified weights of supplier, manufacturer and retailer, respectively, which can reflect their importance in the supply chain, or their power to influence the supply chain.

We name the model structure above the "backward non-cooperative structure", because the evaluation order is the same with the goods flow sequence in a supply chain. The evaluation order also can be reversed, and we call it the "forward non-cooperative structure". In this structure, the retailer is first evaluated, the manufacturer is second evaluated using the information related to retailer's efficiency, and the supplier is third evaluated using both of the retailer's and manufacturer's information about efficiencies. The models of forward non-cooperative structure are similar to the models of backward non-cooperative structure and are omitted here.

3. The partial-cooperative multistage supply chain models

In the backward non-cooperative structure, the first evaluating member has control over the second and the third evaluating ones, the second evaluating member has partial control over only the third one, and the third evaluating member has no control over any other ones. In this section, we propose the partial-cooperative models, in which an alliance is constructed between supplier and manufacturer, or manufacturer and retailer. The alliance is first evaluated, and then with the alliance's information about efficiency which is defined as the arithmetic mean of the alliance members' efficiencies, the supplier or retailer is evaluated.

Suppose supplier and manufacturer are allied and seek to simultaneously maximize both of their efficiencies first. The joint efficiency of the alliance is measured in model (8).

$$
\begin{aligned}
&\max \left[w_A \frac{\sum_r u_r^{A1} y_{rj_0}^{A1} + \sum_r u_r^{A2} y_{rj_0}^{A2}}{\sum_i v_i^A x_{ij_0}^A} + w_B \frac{\sum_r u_r^{B1} y_{rj_0}^{B1} + \sum_r u_r^{B2} y_{rj_0}^{B2}}{\sum_i v_i^B x_{ij_0}^B + \sum_r u_r^{A1} y_{rj_0}^{A1}} \right] = \theta_{(AB)} \\
&s.t.\quad \frac{\sum_r u_r^{A1} y_{rj}^{A1} + \sum_r u_r^{A2} y_{rj}^{A2}}{\sum_i v_i^A x_{ij}^A} \le 1, \\
&\qquad \frac{\sum_r u_r^{B1} y_{rj}^{B1} + \sum_r u_r^{B2} y_{rj}^{B2}}{\sum_i v_i^B x_{ij}^B + \sum_r u_r^{A1} y_{rj}^{A1}} \le 1, \\
&\qquad u_r^{A1}, u_r^{A2}, v_i^A, u_r^{B1}, u_r^{B2}, v_i^B \ge 0,\ j = 1, 2, \ldots, n
\end{aligned} \tag{8}
$$

where $w_A + w_B = 1$, and w_A, w_B are decision maker specified weights of supplier and manufacturer, respectively. The weights indicate the importance or the power of influence of each alliance member in the supply chain, which are not the optimization variables of the program. Chen, Cook, Li and Zhu (2009a) point out that model (8) cannot be turned into a linear program using the usual Charnes and Cooper transformation (Charnes and Cooper, 1962). To solve the problem, they argue that the weights should reflect the "size" of each DMU sub stage, and one reasonable presentation of the "size" is the portion of the total resources devoted to each stage. Based upon their weight choice approach to convert model (8) into a linear program, w_A and w_B can be defined in (9):

$$w_A = \frac{v_i^A x_{ij_0}^A}{v_i^A x_{ij_0}^A + v_i^B x_{ij_0}^B + u_r^{A1} y_{rj_0}^{A1}},\; w_B = \frac{v_i^B x_{ij_0}^B + u_r^{A1} y_{rj_0}^{A1}}{v_i^A x_{ij_0}^A + v_i^B x_{ij_0}^B + u_r^{A1} y_{rj_0}^{A1}}. \tag{9}$$

Then, the objective function of model (8) can be transformed into (10).

$$\frac{\sum_r u_r^{A1} y_{rj_0}^{A1} + \sum_r u_r^{A2} y_{rj_0}^{A2} + \sum_r u_r^{B1} y_{rj_0}^{B1} + \sum_r u_r^{B2} y_{rj_0}^{B2}}{\sum_i v_i^A x_{ij_0}^A + \sum_i v_i^B x_{ij_0}^B + \sum_r u_r^{A1} y_{rj_0}^{A1}}. \tag{10}$$

The optimal solution of model (8) are $u_r^{A1*}, u_r^{A2*}, v_i^{A*}, u_r^{B1*}, u_r^{B2*}$, and v_i^{B*}, and the optimal value $\theta_{(AB)}^*$ is the alliance's efficiency. w_A^* and w_B^* represent the optimal weights obtained from model (8) by through (9). The alternative standard DEA multiplier model of (8) is following:

$$\begin{aligned}
\max & \sum_r \mu_r^{A1} y_{rj_0}^{A1} + \sum_r \mu_r^{A2} y_{rj_0}^{A2} + \sum_r \mu_r^{B1} y_{rj_0}^{B1} + \sum_r \mu_r^{B2} y_{rj_0}^{B2} = \theta_{(AB)} \\
s.t. & \sum_r \mu_r^{A1} y_{rj}^{A1} + \sum_r \mu_r^{A2} y_{rj}^{A2} - \sum_i \omega_i^A x_{ij}^A \le 1, \\
& \sum_r \mu_r^{B1} y_{rj}^{B1} + \sum_r \mu_r^{B2} y_{rj}^{B2} - \sum_i \omega_i^B x_{ij}^B - \sum_r \mu_r^{A1} y_{rj}^{A1} \le 1, \\
& \sum_i \omega_i^A x_{ij_0}^A + \sum_i \omega_i^B x_{ij_0}^B + \sum_r \mu_r^{A1} y_{rj_0}^{A1} = 1, \\
& \mu_r^{A1}, \mu_r^{A2}, \omega_i^A, \mu_r^{B1}, \mu_r^{B2}, \omega_i^B \ge 0,\; j = 1, 2, ..., n.
\end{aligned} \tag{11}$$

Then, the efficiency of the retailer is measured in model (12).

$$\begin{aligned}
\max & \frac{\sum_r u_r^C y_{rj_0}^C}{\sum_i v_i^C x_{ij_0}^C + \sum_r u_r^{B1} y_{rj_0}^{B1}} = \theta_{(AB)C} \\
s.t. & \frac{\sum_r u_r^C y_{rj}^C}{\sum_i v_i^C x_{ij}^C + \sum_r u_r^{B1} y_{rj}^{B1}} \le 1, \\
& \frac{\sum_r u_r^{A1} y_{rj_0}^{A1} + \sum_r u_r^{A2} y_{rj_0}^{A2} + \sum_r u_r^{B1} y_{rj_0}^{B1} + \sum_r u_r^{B2} y_{rj_0}^{B2}}{\sum_i v_i^A x_{ij_0}^A + \sum_i v_i^B x_{ij_0}^B + \sum_r u_r^{A1} y_{rj_0}^{A1}} = \theta_{(AB)}^*, \\
& \frac{\sum_r u_r^{A1} y_{rj}^{A1} + \sum_r u_r^{A2} y_{rj}^{A2}}{\sum_i v_i^A x_{ij}^A} \le 1, \\
& \frac{\sum_r u_r^{B1} y_{rj}^{B1} + \sum_r u_r^{B2} y_{rj}^{B2}}{\sum_i v_i^B x_{ij}^B + \sum_r u_r^{A1} y_{rj}^{A1}} \le 1, \\
& u_r^{A1}, u_r^{A2}, v_i^A, u_r^{B1}, u_r^{B2}, v_i^B, u_r^C, v_i^C \ge 0,\; j = 1, 2, ..., n.
\end{aligned} \tag{12}$$

In model (12), the retailer's efficiency is measured under the condition that the alliance's efficiency remains at $\theta_{(AB)}^*$. The optimal solution of model (9) are $u_r^{A1*}, u_r^{A2*}, v_i^{A*}, u_r^{B1*}, u_r^{B2*}, v_i^{B*}, u_r^{C*}$, and v_i^{C*}, and the optimal value $\theta_{(AB)C}^*$ is the retailer's efficiency when the alliance has first achieved its best performance. The alternative standard DEA multiplier model of (12) is following:

$$
\begin{aligned}
\max & \sum\nolimits_r \mu_r^C y_{rj_0}^C = \theta_{(AB)C} \\
s.t. \quad & \sum\nolimits_r \mu_r^C y_{rj}^C - \sum\nolimits_i \omega_i^C x_{ij}^C - \sum\nolimits_r \mu_r^{B1} y_{rj}^{B1} \le 0, \\
& \sum\nolimits_i \omega_i^C x_{ij_0}^C + \sum\nolimits_r \mu_r^{B1} y_{rj_0}^{B1} = 1, \\
& \sum\nolimits_r \mu_r^{A1} y_{rj_0}^{A1} + \sum\nolimits_r \mu_r^{A2} y_{rj_0}^{A2} + \sum\nolimits_r \dot{\mu}_r^{B1} y_{rj_0}^{B1} + \sum\nolimits_r \mu_r^{B2} y_{rj_0}^{B2} = \theta_{(AB)}^*, \\
& \sum\nolimits_r \mu_r^{A1} y_{rj}^{A1} + \sum\nolimits_r \mu_r^{A2} y_{rj}^{A2} - \sum\nolimits_i \omega_i^A x_{ij}^A \le 0, \\
& \sum\nolimits_r \dot{\mu}_r^{B1} y_{rj}^{B1} + \sum\nolimits_r \mu_r^{B2} y_{rj}^{B2} - \sum\nolimits_i \omega_i^B x_{ij}^B - \sum\nolimits_r \mu_r^{A1} y_{rj}^{A1} \le 0, \\
& \sum\nolimits_i \omega_i^A x_{ij_0}^A + \sum\nolimits_i \omega_i^B x_{ij_0}^B + \sum\nolimits_r \mu_r^{A1} y_{rj_0}^{A1} = 1 \\
& \mu_r^{A1}, \mu_r^{A2}, \omega_i^A, \mu_r^{B1}, \dot{\mu}_r^{B1}, \mu_r^{B2}, \omega_i^B, \mu_r^C, \omega_i^C \ge 0,\ j = 1, 2, \ldots, n.
\end{aligned}
\tag{13}
$$

Each alliance member’s efficiency also can be measured individually by using the optimal solution of model (8) as in (14).

$$
\theta_{A/(AB)}^* = \frac{\sum_r u_r^{A1*} y_{rj}^{A1} + \sum_r u_r^{A2*} y_{rj}^{A2}}{\sum_i v_i^{A*} x_{ij}^A}, \theta_{B/(AB)}^* = \frac{\sum_r u_r^{B1*} y_{rj}^{B1} + \sum_r u_r^{B2*} y_{rj}^{B2}}{\sum_i v_i^{B*} x_{ij}^B + \sum_r u_r^{A1*} y_{rj}^{A1}}. \tag{14}
$$

Considering the definition in section 3, the efficiency of the whole supply chain can be defined as the arithmetic mean of the alliance’s and retailer’s efficiencies as follow:

$$
E_{(AB)C} = w_1 \theta_{(AB)} + w_2 \theta_{(AB)C}, \text{ and } \ w_1 + w_2 = 1. \tag{15}
$$

where w_1 and w_2 are decision maker specified weights.

The alliance also can be built up between the manufacture and retailer, and the efficiency of the alliance is first measured. Then, the efficiency of the supplier is measured with the alliance’s information of efficiency. The models under this supply chain structure are similar to model (8) and (12) and are omitted here.

4. The cooperative multistage supply chain models

In this section, we will consider the cooperative structure of supply chain, in which all of the members are evaluated simultaneously. The cooperative model seeks to maximize the joint efficiency defined as the arithmetic mean of supplier’s, manufacturer’s and retailer’s efficiencies in model (12).

$$
\begin{aligned}
\max\ & w_A \frac{\sum_r u_r^{A1} y_{rj_0}^{A1} + \sum_r u_r^{A2} y_{rj_0}^{A2}}{\sum_i v_i^A x_{ij_0}^A} + w_B \frac{\sum_r u_r^{B1} y_{rj_0}^{B1} + \sum_r u_r^{B2} y_{rj_0}^{B2}}{\sum_i v_i^B x_{ij_0}^B + \sum_r u_r^{A1} y_{rj_0}^{A1}} + w_C \frac{\sum_r u_r^C y_{rj_0}^C}{\sum_i v_i^C x_{ij_0}^C + \sum_r u_r^{B1} y_{rj_0}^{B1}} = \theta_{(ABC)} \\
s.t.\ & \frac{\sum_r u_r^{A1} y_{rj}^{A1} + \sum_r u_r^{A2} y_{rj}^{A2}}{\sum_i v_i^A x_{ij}^A} \le 1, \\
& \frac{\sum_r u_r^{B1} y_{rj}^{B1} + \sum_r u_r^{B2} y_{rj}^{B2}}{\sum_i v_i^B x_{ij}^B + \sum_r u_r^{A1} y_{rj}^{A1}} \le 1, \\
& \frac{\sum_r u_r^C y_{rj}^C}{\sum_i v_i^C x_{ij}^C + \sum_r u_r^{B1} y_{rj}^{B1}} \le 1, \\
& w_A + w_B + w_C = 1, \\
& u_r^{A1}, u_r^{A2}, v_i^A, u_r^{B1}, u_r^{B2}, v_i^B, u_r^C, v_i^C \ge 0,\ j = 1, 2, \ldots, n.
\end{aligned}
\tag{16}
$$

where w_A, w_B and w_C are also decision maker specified weights for supplier, manufacturer and retailer, respectively, and reflect the importance or the power of influence of each member in the supply chain. Each weight also can be defined according to the portion of the total resources devoted to each member as in (17).

$$w_A = \frac{\sum_i v_i^A x_{ij_0}^A}{\sum_i v_i^A x_{ij_0}^A + \sum_i v_i^B x_{ij_0}^B + \sum_i v_i^C x_{ij_0}^C + \sum_r u_r^{A1} y_{rj_0}^{A1} + \sum_r u_r^{B1} y_{rj_0}^{B1}},$$
$$w_B = \frac{\sum_i v_i^B x_{ij_0}^B + \sum_r u_r^{A1} y_{rj_0}^{A1}}{\sum_i v_i^A x_{ij_0}^A + \sum_i v_i^B x_{ij_0}^B + \sum_i v_i^C x_{ij_0}^C + \sum_r u_r^{A1} y_{rj_0}^{A1} + \sum_r u_r^{B1} y_{rj_0}^{B1}}, \tag{17}$$
$$w_C = \frac{\sum_i v_i^C x_{ij_0}^C + \sum_r u_r^{B1} y_{rj_0}^{B1}}{\sum_i v_i^A x_{ij_0}^A + \sum_i v_i^B x_{ij_0}^B + \sum_i v_i^C x_{ij_0}^C + \sum_r u_r^{A1} y_{rj_0}^{A1} + \sum_r u_r^{B1} y_{rj_0}^{B1}}.$$

The optimal solution of model (12) are $u_r^{A1*}, u_r^{A2*}, v_i^{A*}, u_r^{B1*}, u_r^{B2*}, v_i^{B*}, u_r^{C*}$, and v_i^{C*}, and the optimal value θ_{ABC}^* is the efficiency of the whole supply chain. The alternative standard DEA multiplier model of (16) is following:

$$\begin{aligned}
\max\ & \sum_r u_r^{A1} y_{rj_0}^{A1} + \sum_r u_r^{A2} y_{rj_0}^{A2} + \sum_r u_r^{B1} y_{rj_0}^{B1} + \sum_r u_r^{B2} y_{rj_0}^{B2} + \sum_r u_r^{C} y_{rj_0}^{C} = \theta_{(ABC)} \\
s.t.\ & \sum_r u_r^{A1} y_{rj}^{A1} + \sum_r u_r^{A2} y_{rj}^{A2} - \sum_i v_i^A x_{ij}^A \le 0, \\
& \sum_r u_r^{B1} y_{rj}^{B1} + \sum_r u_r^{B2} y_{rj}^{B2} - \sum_i v_i^B x_{ij}^B - \sum_r u_r^{A1} y_{rj}^{A1} \le 0, \\
& \sum_r u_r^{C} y_{rj}^{C} - \sum_i v_i^C x_{ij}^C - \sum_r u_r^{B1} y_{rj}^{B1} \le 0, \\
& \sum_i v_i^A x_{ij_0}^A + \sum_i v_i^B x_{ij_0}^B + \sum_r u_r^{A1} y_{rj_0}^{A1} + \sum_i v_i^C x_{ij_0}^C + \sum_r u_r^{B1} y_{rj_0}^{B1} = 1, \\
& u_r^{A1}, u_r^{A2}, v_i^A, u_r^{B1}, u_r^{B2}, v_i^B, u_r^C, v_i^C \ge 0,\ j = 1, 2, \ldots, n.
\end{aligned} \tag{18}$$

By using the optimal solution of model (16), each supply chain member's efficiency could be measured individually in (19).

$$\theta_{A/(ABC)}^* = \frac{\sum_r u_r^{A1*} y_{rj}^{A1} + \sum_r u_r^{A2*} y_{rj}^{A2}}{\sum_i v_i^{A*} x_{ij}^A},$$
$$\theta_{B/(ABC)}^* = \frac{\sum_r u_r^{B1*} y_{rj}^{B1} + \sum_r u_r^{B2*} y_{rj}^{B2}}{\sum_i v_i^{B*} x_{ij}^B + \sum_r u_r^{A1*} y_{rj}^{A1}}, \tag{19}$$
$$\theta_{C/(ABC)}^* = \frac{\sum_r u_r^{C*} y_{rj}^{C}}{\sum_i v_i^{C*} x_{ij}^C + \sum_r u_r^{B1*} y_{rj}^{B1}}.$$

5. The general frameworks for multistage supply chain models

Now, we consider the extension to the general frameworks for multistage supply chain with P stages under the non-cooperative and the cooperative structures.

Consider a P stage supply chain illustrated in Figure 2. The input of stage 1 is denoted by M^0. There are two forms of outputs from stage p (p=1,…,P) which is $M^{p(1)}$ and $M^{p(2)}$. The former represents the output that leaves this stage and will not become the input to the p+1 stage, and the later represents the output that becomes the input to the next stage. Furthermore, there are two forms of inputs to stage p. $M^{p\text{-}1(2)}$ represent the intermediate measure from stage p-1, and $M^{p\text{-}1(3)}$ represent a new input enter the supply chain at stage p.

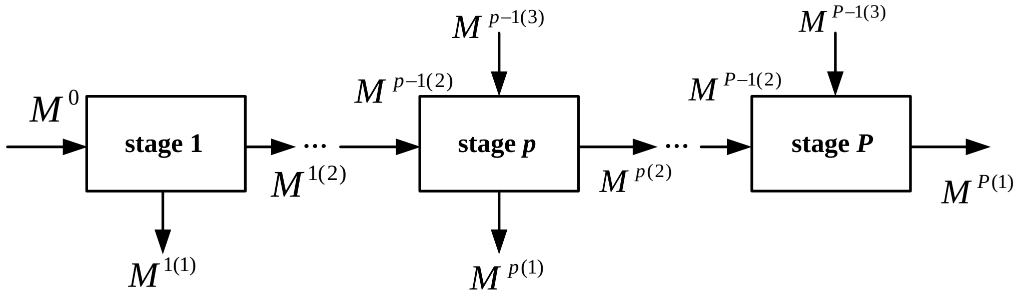

Figure 2. Multistage supply chain structure

When p=2,…,P, $m_{rj}^{p(1)}$ (r=1,…,R) is the rth component of output from stage p of DMU$_j$, that leaves the supply chain at this stage and will not become the input to stage p+1; $m_{kj}^{p(2)}$ (k=1,…,K) is the kth component of output from stage p of DMU$_j$, that becomes the input to stage p+1; and $m_{ij}^{p-1(3)}$ (i=1,…,I) is the ith component of input to stage p of DMU$_j$, that enters the supply chain at this stage. At the first stage 1, all inputs are denoted as m_{ij}^{0}, and at the last stage P, all outputs are seen as $m_{rj}^{P(1)}$. u_r^p, w_k^p, and v_i^{p-1} are the weights associated with outputs $m_{rj}^{p(1)}, m_{kj}^{p(2)}$, and $m_{ij}^{p-1(3)}$, respectively.

Therefore, when p=2,…,P, the general model for the efficiency measure of stage p in a multistage supply chain with P stages under the non-cooperative structure could be expressed as in model (20):

$$
\begin{aligned}
\max\ & \frac{\sum_r u_r^p m_{rj_0}^{p(1)} + \sum_k w_k^p m_{kj_0}^{p(2)}}{\sum_k w_k^{p-1} m_{kj_0}^{p-1(2)} + \sum_i v_i^{p-1} m_{ij_0}^{p-1(3)}} = \theta_p \\
s.t.\ & \frac{\sum_r u_r^p m_{rj}^{p(1)} + \sum_k w_k^p m_{kj}^{p(2)}}{\sum_k w_k^{p-1} m_{kj}^{p-1(2)} + \sum_i v_i^{p-1} m_{ij}^{p-1(3)}} \le 1, \\
& \frac{\sum_r u_r^q m_{rj_0}^{q(1)} + \sum_k u_k^q m_{rj_0}^{q(2)}}{\sum_k w_k^{q-1} m_{kj_0}^{q-1(2)} + \sum_i v_i^{q-1} m_{ij_0}^{q-1(3)}} = \theta_q^*, \\
& \frac{\sum_r u_r^q m_{rj}^{q(1)} + \sum_k u_k^q m_{rj}^{q(2)}}{\sum_k w_k^{q-1} m_{kj}^{q-1(2)} + \sum_i v_i^{q-1} m_{ij}^{q-1(3)}} \le 1, \\
& u_r^p, w_k^p, v_i^p \ge 0, r=1,\dots,R, k=1,\dots,K, i=1,\dots,I, \\
& j=1,\dots,n, p=1,\dots,P, q=1,\dots,p-1.
\end{aligned}
\tag{20}
$$

where θ_p is the efficiency of stage p which is measured under the condition that the efficiencies of stage 1 to stage q(=p-1) remain at θ_q^*.

Similar to Cook, Zhu, Bi and Yang (2010), the general model for the overall efficiency measure of multistage supply chain with P stages under the cooperative structure could be expressed as in model (21):

$$\max \frac{\sum_p \left(\sum_r u_r^p m_{rj_0}^{p(1)} + \sum_k w_k^p m_{kj_0}^{p(2)}\right)}{\sum_i v_i^0 m_{ij_0}^0 + \sum_p \left(\sum_k w_k^{p-1} m_{kj_0}^{p-1(2)} + \sum_i v_i^{p-1} m_{ij_0}^{p-1(3)}\right)} = \theta_{overall}$$

$$\begin{aligned} s.t.\quad & \frac{\sum_r u_r^1 m_{rj}^{1(1)} + \sum_k w_k^1 m_{kj}^{1(2)}}{\sum_i v_i^0 m_{ij}^0} \le 1, \\ & \frac{\sum_r u_r^p m_{rj}^{p(1)} + \sum_k w_k^p m_{kj}^{p(2)}}{\sum_k w_k^{p-1} m_{kj}^{p-1(2)} + \sum_i v_i^{p-1} y_{ij}^{p-1(3)}} \le 1, \\ & u_r^p, w_k^p, v_i^p \ge 0, r=1,\dots,R, k=1,\dots,K, i=1,\dots,I, p=1,\dots,P, j=1,\dots,n. \end{aligned} \tag{21}$$

where m_{ij}^0 and v_i^0 are the input and associated weight for stage 1, and $\theta_{overall}$ is the overall efficiency of the multistage supply chain.

6. An illustrative example

We now apply the proposed multistage supply chain DEA models to the numerical example used in Tone and Tsutsui (2009). Table 1 provides the data for the efficiency measurement of ten vertically integrated electric power companies consist of three divisions of power generation, transmission and distribution. The inputs, outputs and intermediate measures are as follows. The input of the power generation stage is x^A (labor), and the output of this stage is y^A (electric power generated), which is also the intermediate input of the power transmission stage. The transmission stage have another input x^B (labor), one output y^{B2} (electric power sold to large customers) and one intermediate output y^{B1} (electric power sent), which is also the intermediate input of the power distribution stage. Another input of the power distribution stage is x^c (labor), and the output of this stage is y^C (electric power sold to small customers).

Table 2 reports the efficiency scores obtained from the black box model, backward non-cooperative model, partial-cooperative model, and cooperative model. We first solve the black box model (1) using input x^A, x^B and x^c, and output y^{B2} and y^C, where the intermediate input and output are neglected. The black box overall efficiency of the supply chain is reported in Table 2, and half of the DMUs are measured as overall efficient. We then solve non-cooperative model (2), (4), and (6), partial-cooperative model (11) and (13), and cooperative model (18). E-ABC, E-(AB)C and E-(ABC) are the overall efficiency scores of the supply chain under these three different models, respectively. We point out that in the backward non-cooperative model, when calculating the overall efficiency, the weights assigned to the power generation, transmission and distribution stages are 0.4, 0.2 and 0.4, respectively, and these weights selection are just for illustrative purpose (same with Tone and Tsutsui's (2009) example). Furthermore, in the partial-cooperative model, the weights assigned to the alliance of power generation and transmission stages, and the power distribution stage are 0.6 and 0.4, respectively. No supply chain is measured efficient under the non-cooperative and partial-cooperative model, and only one supply chain (DMU 3) achieves efficient under cooperation model.

Table 2 also reports that, under the backward non-cooperative structure, no DMU is rated as overall efficient. However, the power generation and transmission stages for DMU 3, the power distribution stage for DMU 4, and the power transmission stage for DMU 9 are measured as efficient. Similar evaluation results also can be found under the partial-cooperative and cooperative models.

The efficiency scores are compared in Figure 3. It can be seen that the scores of the black box model tend to be higher than the scores of cooperation model, and the scores of non-cooperative model are the lowest. This figure clearly indicates that, in this example, the multistage DEA models have higher discriminate power than that of the black box DEA model. Figure 3 also shows that the trends of the efficiency scores of three multistage DEA models are roughly similar, and on average, the efficiency under the cooperative model appears the highest and the efficiency under the non-cooperative model lowest.

Table 1. Data of inputs and outputs for the multistage supply chain

DMU	x^A	y^{A1}	x^B	y^{B2}	y^{B1}	x^C	y^C
1	0.838	0.894	0.277	0.879	0.362	0.962	0.337
2	1.233	0.678	0.132	0.538	0.188	0.443	0.18
3	0.321	0.836	0.045	0.911	0.207	0.482	0.198
4	1.483	0.869	0.111	0.57	0.516	0.467	0.491
5	1.592	0.693	0.208	1.086	0.407	1.073	0.372
6	0.79	0.966	0.139	0.722	0.269	0.545	0.253
7	0.451	0.647	0.075	0.509	0.257	0.366	0.241
8	0.408	0.756	0.074	0.619	0.103	0.229	0.097
9	1.864	1.191	0.061	1.023	0.402	0.691	0.38
10	1.222	0.792	0.149	0.769	0.187	0.337	0.178

Table 2. Data of efficiencies for the multistage supply chain and its members

DMU		1	2	3	4	5	6	7	8	9	10
Black box overall efficiency		0.6999 (8)	0.5870 (9)	1.0000 (3)	1.0000 (3)	0.5748 (10)	0.7621 (7)	1.0000 (3)	1.0000 (3)	1.0000 (3)	0.9530 (6)
Backward non-cooperative structure	E-ABC	0.4988 (7)	0.3513 (10)	0.9997 (1)	0.6783 (4)	0.3714 (9)	0.5326 (6)	0.7134 (2)	0.5765 (5)	0.6941 (3)	0.4141 (8)
	θ_A	0.4096	0.2111	1.0000	0.2250	0.1671	0.4695	0.5508	0.7115	0.2453	0.2489
	θ_{AB}	0.5953	0.3673	1.0000	0.9418	0.5523	0.5587	0.8104	0.6538	1.0000	0.4228
	θ_{ABC}	0.5397	0.4834	0.9992	1.0000	0.4853	0.5826	0.8274	0.4029	0.9900	0.5749
Partial-cooperative structure	E-(AB)C	0.4839 (8)	0.3619 (10)	0.8260 (2)	0.8222 (3)	0.3944 (9)	0.5333 (7)	0.8068 (4)	0.5840 (6)	0.9938 (1)	0.6197 (5)
	$\theta_{(AB)}$	0.5202	0.3155	1.0000	0.7037	0.3851	0.5476	0.6891	0.7047	0.9963	0.3684
	$\theta_{A/(AB)}$	0.4096	0.2111	1.0000	0.2250	0.1671	0.4695	0.5508	0.7115	0.2453	0.2489
	$\theta_{B/(AB)}$	0.6455	0.6019	1.0000	0.7056	0.9874	0.6559	0.8621	0.6966	1.0000	0.7190
	$\theta_{(AB)C}$	0.4295	0.4315	0.5651	1.0000	0.4083	0.5119	0.9833	0.4029	0.9900	0.9965
Cooperative structure	E-(ABC) $=\theta_{(ABC)}$	0.5793 (7)	0.3863 (10)	1.0000 (1)	0.9965 (2)	0.4782 (9)	0.5977 (6)	0.7617 (4)	0.7046 (5)	0.9934 (3)	0.5018 (8)
	$\theta_{A/(ABC)}$	0.4096	0.2111	1.0000	0.2250	0.1671	0.4695	0.5508	0.7115	0.2453	0.2489
	$\theta_{B/(ABC)}$	0.6455	0.5968	1.0000	0.7843	0.5039	0.6559	0.7171	0.6966	1.0000	0.6771
	$\theta_{C/(ABC)}$	0.9711	0.3867	0.9992	1.0000	0.9535	0.9837	0.9833	0.5849	0.9900	0.5025

Note: parentheses under the each efficiency score denote the rank of each DMU

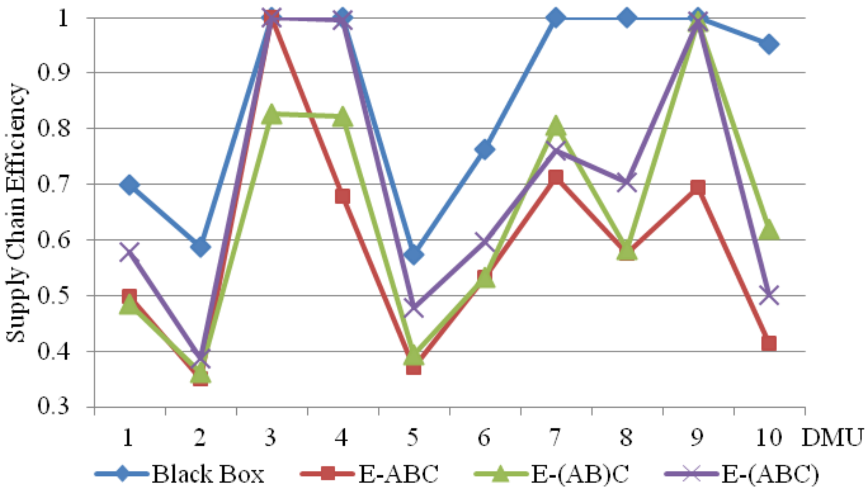

Figure 3. Efficiency of supply chain under different models

7. Conclusion

In this paper, we developed several multistage supply chain DEA models for measuring the efficiencies of the whole supply chain and it members under different supply structures. We consider a representative multistage supply chain which has three members: supplier, manufacturer and retailer. The non-cooperative structure model is first proposed to evaluate the efficiency of each supply chain member according to a specified sequence (backward and forward process). The partial-cooperative structure model is second proposed to evaluate the efficiency of the alliance, which is constructed between two of the supply chain members, and the other member which is not in the alliance. The cooperative structure model is third proposed to evaluate the efficiencies of all of the supply chain members simultaneously. The intermediate measures between each supply chain member are considered which link the members and the whole supply chain. Two different kinds of inputs/outputs (normal and intermediate) are also considered in the models in order to make the multistage supply chain models becoming more general. Although some of these models are nonlinear programming problems, they can be solved as linear programming problems by choosing decision maker specified weights. Moreover, we extend these models to the general frameworks for multistage supply chain with P stages under the non-cooperative and the cooperative structures.

Because traditional DEA models cannot be directly applied to evaluating multistage supply chain, the models proposed in this paper could be seen as efficient tools for the performance evaluation of this kind of supply chain. Although in this study, we use the cooperative and non-cooperative concepts for modeling the multistage supply china, we are not trying to examine whether the supplier, manufacture, and retailer of a specific supply chain are behaving in a cooperative, non-cooperative or partial-cooperative manner.

Also, we have to point out that the current study is not an empirical research, and we only propose an analytical framework for measuring the efficiency of multistage supply chain. Therefore, an example of efficiency measurement of vertically integrated electric power companies is used to demonstrate our work of current paper. The example illustrates that the multistage models could provide more detailed information about the efficiency or inefficiency decomposition to each supply member. Therefore, these models may become useful tools for the managers in monitoring and planning their supply chain operations so as to significantly aid them making multistage supply chains more efficient.

While the major work of the current study lies in the methodological developments, it has potential for application in various supply chain operations when enough related evaluation data do exist or can be acquired. Finally, we point out that the models proposed in this paper are based on constant returns to scale (CRS) DEA. The variable returns to scale (VRS) DEA for multistage supply chain performance evaluation is a subject for future research. Furthermore the performance evaluation of specific multistage supply chains which have similar structures proposed in our models is another subject for further research.

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